THE DIGITAL SAT

MATH

QUESTION TYPES & STRATEGIES

PLUG IN & TEST or PICK NUMBERS

Question 1

$$y \leq 3x + 1$$
$$x - y > 1$$

Which of the following ordered pairs (x, y) satisfies the system of inequalities above?

A) $(-2, -1)$

B) $(-1, 3)$

C) $(1, 5)$

D) $(2, -1)$

Question 2

Which of the following is equivalent to $\dfrac{4x^2 + 6x}{4x + 2}$?

A) x

B) $x + 4$

C) $x - \dfrac{2}{4x + 2}$

D) $x + 1 - \dfrac{2}{4x + 2}$

Question 3

$$f(x) = 5{,}470(0.64)^{\frac{x}{12}}$$

The function f gives the value, in dollars, of a certain piece of equipment after x months of use. If the value of the equipment decreases each year by p% of its value the preceding year, what is the value of p ?

A) 4

B) 5

C) 36

D) 64

Question 4

Which of the following is equivalent to the expression $x^4 - x^2 - 6$?

A. $(x^2+1)(x^2-6)$

B. $(x^2+2)(x^2-3)$

C. $(x^2+3)(x^2-2)$

D. $(x^2+6)(x^2-1)$

Question 5

What is the set of all solutions to the equation $\sqrt{x+2} = -x$?

A) $\{-1, 2\}$

B) $\{-1\}$

C) $\{2\}$

D) There are no solutions to the given equation.

Question 6

The function f is defined by $f(x) = a\sqrt{x+b}$, where a and b are constants. In the xy-plane, the graph of $y = f(x)$ passes through the point $(-24, 0)$, and $f(24) < 0$. Which of the following must be true?

(A) $f(0) = 24$ (A)

(B) $f(0) = -24$ (B)

(C) $a > b$ (C)

(D) $a < b$ (D)

Question 7

$$7x + 6y = 5$$

$$28x + 24y = 20$$

For each real number r, which of the following points lies on the graph of each equation in the *xy*-plane for the given system?

A. $\left(r, -\frac{6r}{7} + \frac{5}{7}\right)$

B. $\left(r, \frac{7r}{6} + \frac{5}{6}\right)$

C. $\left(\frac{r}{4} + 5, -\frac{r}{4} + 20\right)$

D. $\left(-\frac{6r}{7} + \frac{5}{7}, r\right)$

Question 8

$$F(x) = \frac{9}{5}(x - 273.15) + 32$$

The function F gives the temperature, in degrees Fahrenheit, that corresponds to a temperature of x kelvins. If a temperature increased by 9.10 kelvins, by how much did the temperature increase, in degrees Fahrenheit?

A. **16.38**

B. **48.38**

C. **475.29**

D. **507.29**

Linear Equations/Linear Inequalities/Systems of Linear Equations

What is slope-intercept form?

Slope-intercept is a specific form of linear equations. It has the following general structure.

$$y = mx + b$$

m is the slope, which is the the rise of a line over the run of a line, or the change in the vertical direction (y) over the change in the horizontal direction (x).

Formula

Slope of a line

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

m = slope

(x_1, y_1) = coordinates of first point in the line

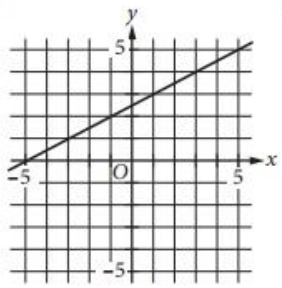
(x_2, y_2) = coordinates of second point in the line

The meaning of Slope: The slope of a line represents how much the y-value changes for every **one-unit** increase in the x-value.

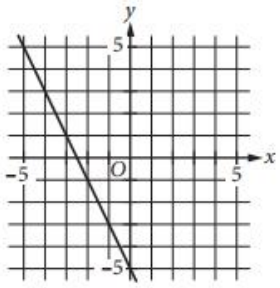
Question 9

Which of the following is the graph of the equation $y = 2x - 5$ in the xy-plane?

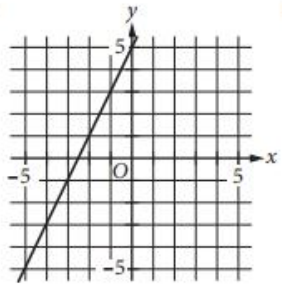
A)



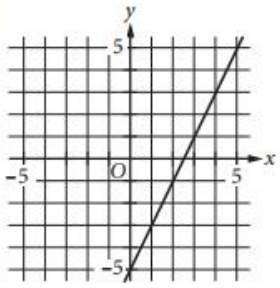
B)



C)



D)



Question 10

Line t in the xy-plane has a slope of $-\frac{1}{3}$ and passes through the point $(9, 10)$. Which equation defines line t ?

A) $y = 13x - \frac{1}{3}$

B) $y = 9x + 10$

C) $y = -\frac{x}{3} + 10$

D) $y = -\frac{x}{3} + 13$

Question 11

$$-2x + 3y = 6$$

In the xy-plane, the graph of which of the following equations is perpendicular to the graph of the equation above?

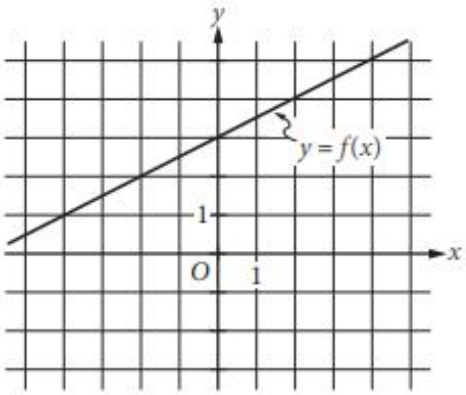
A) $3x + 2y = 6$

B) $3x + 4y = 6$

C) $2x + 4y = 6$

D) $2x + 6y = 3$

Question 12



The graph of the linear function f is shown in the xy-plane above. The slope of the graph of the linear function g is 4 times the slope of the graph of f. If the graph of g passes through the point $(0, -4)$, what is the value of $g(9)$?

A) 5

B) 9

C) 14

D) 18

Question 13

Which of the following equations represents a line that is parallel to the line with equation $y = -3x + 4$?

A) $6x + 2y = 15$

B) $3x - y = 7$

C) $2x - 3y = 6$

D) $x + 3y = 1$

Question 14

$$kx - 3y = 4$$
$$4x - 5y = 7$$

In the system of equations above, k is a constant and x and y are variables. For what value of k will the system of equations have no solution?

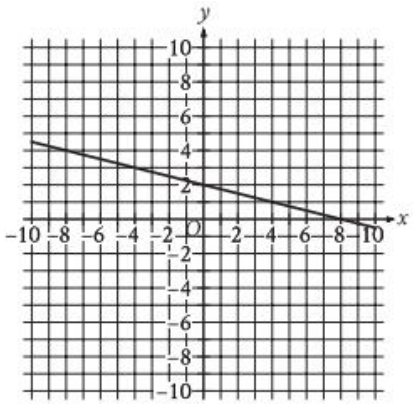
A) $\frac{12}{5}$

B) $\frac{16}{7}$

C) $-\frac{16}{7}$

D) $-\frac{12}{5}$

Question 15



The graph of $y = f(x) + 14$ is shown. Which equation defines function f?

A) $f(x) = -\frac{1}{4}x - 12$

B) $f(x) = -\frac{1}{4}x + 16$

C) $f(x) = -\frac{1}{4}x + 2$

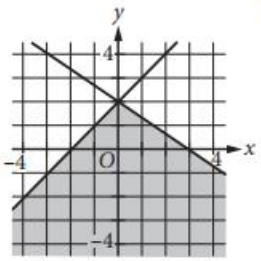
D) $f(x) = -\frac{1}{4}x - 14$

Question 16

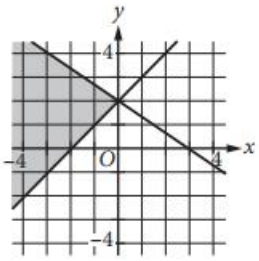
$$y \geq x + 2$$
$$2x + 3y \leq 6$$

In which of the following does the shaded region represent the solution set in the xy-plane to the system of inequalities above?

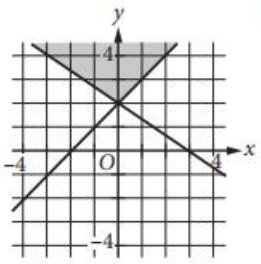
A)



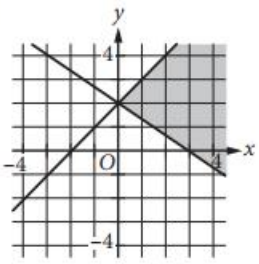
B)



C)

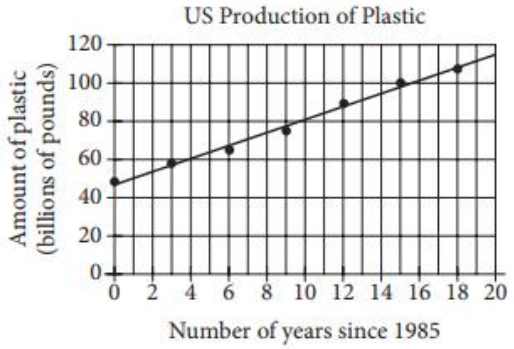


D)



Question 17

Between 1985 and 2003, data were collected every three years on the amount of plastic produced annually in the United States, in billions of pounds. The graph below shows the data and a line of best fit. The equation of the line of best fit is $y = 3.39x + 46.89$, where x is the number of years since 1985 and y is the amount of plastic produced annually, in billions of pounds.



Which of the following is the best interpretation of the number 3.39 in the context of the problem?

A) The amount of plastic, in billions of pounds, produced in the United States during the year 1985

B) The number of years it took the United States to produce 1 billion pounds of plastic

C) The average annual plastic production, in billions of pounds, in the United States from 1985 to 2003

D) The average annual increase, in billions of pounds, of plastic produced per year in the United States from 1985 to 2003

Math Vocabulary

WHEN YOU SEE:	THINK:
sum, plus, more than, added to, combined total	+
minus, less than, difference between, decreased by	−
is, was, equals, is equivalent to, is the same as, adds up to	=
times, product, multiplied by, of, twice, double, triple	×
divided by, over, quotient, per, out of, into	÷
what, how much, how many, a number	*x*, *n*, etc.

Now, try translating the following five phrases from English to math.

English	Math
1. *y* is 5 more than *x*.	_____
2. *r* equals half of *s*.	_____
3. *x* is twice as great as *y*.	_____
4. 2 less than *m* is equal to *n*.	_____
5. The product of *a* and *b* is 3 more than their sum.	_____
6. He bought twice as many apples as oranges.	_____
7. m is at least twice as much as n.	_____
8. a is at most three times as much as b.	_____

Question 18

A laundry service is buying detergent and fabric softener from its supplier. The supplier will deliver no more than 300 pounds in a shipment. Each container of detergent weighs 7.35 pounds, and each container of fabric softener weighs 6.2 pounds. The service wants to buy at least twice as many containers of detergent as containers of fabric softener. Let d represent the number of containers of detergent, and let s represent the number of containers of fabric softener, where d and s are nonnegative integers. Which of the following systems of inequalities best represents this situation?

A) $7.35d + 6.2s \leq 300$
$d \geq 2s$

B) $7.35d + 6.2s \leq 300$
$2d \geq s$

C) $14.7d + 6.2s \leq 300$
$d \geq 2s$

D) $14.7d + 6.2s \leq 300$
$2d \geq s$

Question 19

A group of friends decided to divide the $800 cost of a trip equally among themselves. When two of the friends decided not to go on the trip, those remaining still divided the $800 cost equally, but each friend's share of the cost increased by $20. How many friends were in the group originally?

QUADRATIC EQUATIONS

THE QUADRATIC FORMULA

If $ax^2 + bx + c = 0$ but $a \neq 0$

then $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

DISCRIMINANT

- $\Rightarrow$ $b^2 - 4ac > 0$ two real solutions
- $\Rightarrow$ $b^2 - 4ac = 0$ one real solutions
- $\Rightarrow$ $b^2 - 4ac < 0$ zero real solutions

The Vertex Form of the Quadratic Equation:

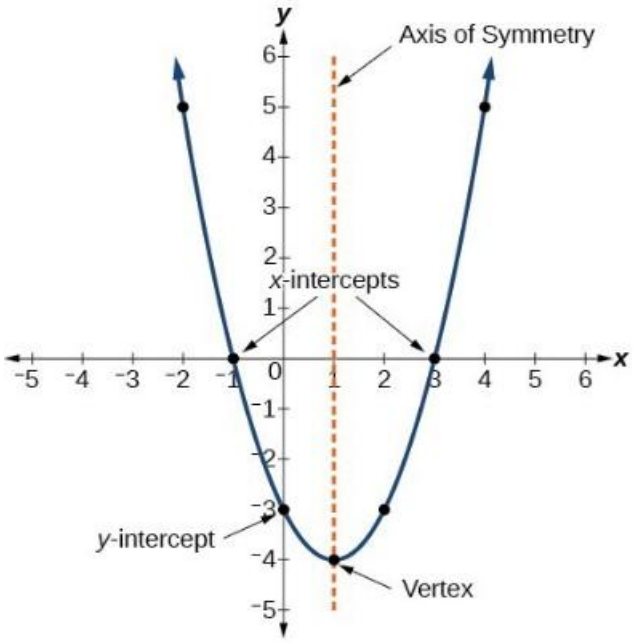
$$y = a(x - h)^2 + k$$

vertex (h, k)

where $h = \frac{-b}{2a}$ and evaluate y at h to find k.

The axis of symmetry:

The axis of symmetry of a parabola is a vertical line that divides the parabola into two symmetrical halves, like a mirror image.



Question 20

What are the solutions to $3x^2 + 12x + 6 = 0$?

A) $x = -2 \pm \sqrt{2}$

B) $x = -2 \pm \frac{\sqrt{30}}{3}$

C) $x = -6 \pm \sqrt{2}$

D) $x = -6 \pm 6\sqrt{2}$

Question 21

The function f is defined by $f(x) = (x + 3)(x + 1)$. The graph of f in the xy-plane is a parabola. Which of the following intervals contains the x-coordinate of the vertex of the graph of f ?

A) $-4 < x < -3$

B) $-3 < x < 1$

C) $1 < x < 3$

D) $3 < x < 4$

Question 22

$$5x^2 + 10x + 16 = 0$$

How many distinct real solutions does the given equation have?

A) Exactly one
B) Exactly two
C) Infinitely many
D) Zero

Question 23

$$x = 2y + 5$$
$$y = (2x - 3)(x + 9)$$

How many ordered pairs (x, y) satisfy the system of equations shown above?

A) 0
B) 1
C) 2
D) Infinitely many

Question 24

If $(ax+2)(bx+7) = 15x^2 + cx + 14$ for all values of x, and $a + b = 8$, what are the two possible values for c ?

A) 3 and 5
B) 6 and 35
C) 10 and 21
D) 31 and 41

Question 25

$$y = -(x-3)^2 + a$$

In the equation above, a is a constant. The graph of the equation in the xy-plane is a parabola. Which of the following is true about the parabola?

A) Its minimum occurs at $(-3, a)$.

B) Its minimum occurs at $(3, a)$.

C) Its maximum occurs at $(-3, a)$.

D) Its maximum occurs at $(3, a)$.

Question 26

$$f(x) = (x+6)(x-4)$$

Which of the following is an equivalent form of the function f above in which the minimum value of f appears as a constant or coefficient?

A) $f(x) = x^2 - 24$

B) $f(x) = x^2 + 2x - 24$

C) $f(x) = (x-1)^2 - 21$

D) $f(x) = (x+1)^2 - 25$

Question 27

$$y = 2x^2 - 21x + 64$$
$$y = 3x + a$$

In the given system of equations, a is a constant. The graphs of the equations in the given system intersect at exactly one point, (x, y), in the xy-plane. What is the value of x ?

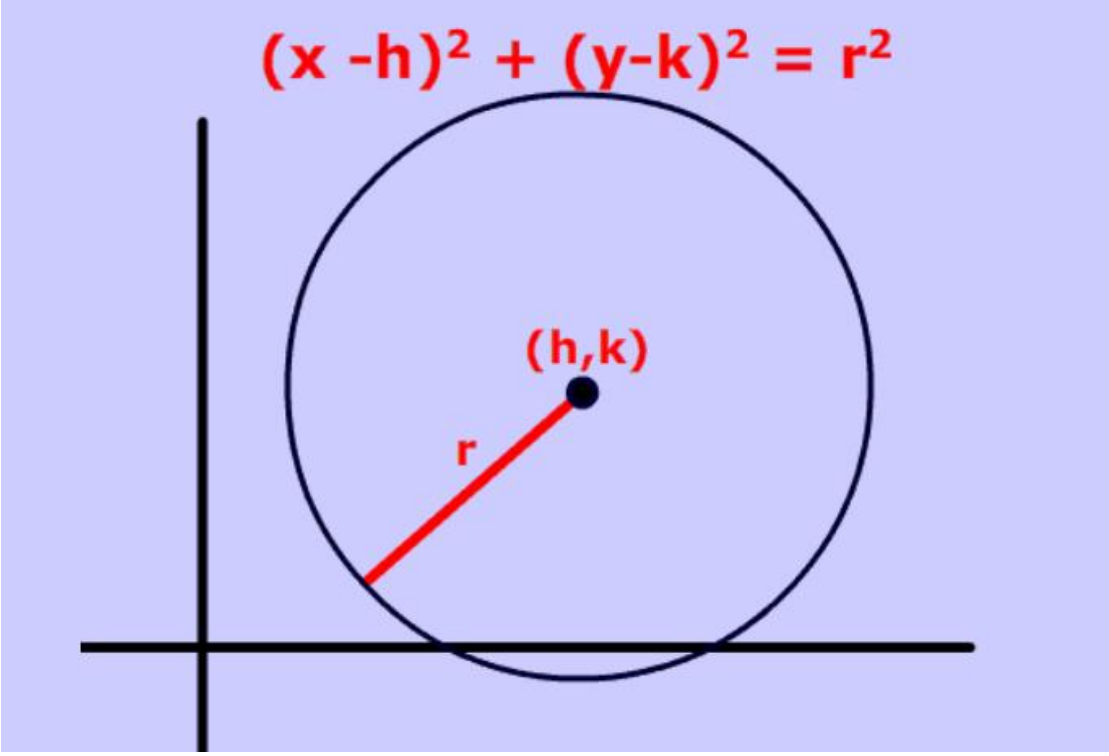
A) -8

B) -6

C) 6

D) 8

Circle Equations



Question 28

A circle in the xy-plane has center $(5,7)$ and radius 2.
Which of the following is an equation of the circle?

A) $(x-5)^2+(y-7)^2=4$

B) $(x+5)^2+(y+7)^2=4$

C) $(x-5)^2+(y-7)^2=2$

D) $(x+5)^2+(y+7)^2=2$

SQUARE OF A SUM

$$(a+b)^2 = a^2 + 2ab + b^2$$

SQUARE OF A DIFFERNCE

$$(a-b)^2 = a^2 - 2ab + b^2$$

Question 29

In the xy-plane, the graph of $2x^2 - 6x + 2y^2 + 2y = 45$ is a circle. What is the radius of the circle?

A) 5

B) 6.5

C) $\sqrt{40}$

D) $\sqrt{50}$

Question 30

A circle in the xy-plane has equation $(x+3)^2 + (y-1)^2 = 25$. Which of the following points does NOT lie in the interior of the circle?

A) $(-7, 3)$

B) $(-3, 1)$

C) $(0, 0)$

D) $(3, 2)$

Question 31

A circle in the *xy*-plane has its center at $(-4, -6)$. Line k is tangent to this circle at the point $(-7, -7)$. What is the slope of line k?

A. -3

B. $-\frac{1}{3}$

C. $\frac{1}{3}$

D. 3

Question 32

In a circle with center O, central angle AOB has a measure of $\frac{5\pi}{4}$ radians. The area of the sector formed by central angle AOB is what fraction of the area of the circle?

Exponents & Roots

Exponent Rules For $a \neq 0, b \neq 0$	
Product Rule	$a^x \times a^y = a^{x+y}$
Quotient Rule	$a^x \div a^y = a^{x-y}$
Power Rule	$\left(a^x\right)^y = a^{xy}$
Power of a Product Rule	$(ab)^x = a^x b^x$
Power of a Fraction Rule	$\left(\frac{a}{b}\right)^x = \frac{a^x}{b^x}$
Zero Exponent	$a^0 = 1$
Negative Exponent	$a^{-x} = \frac{1}{a^x}$
Fractional Exponent	$a^{\frac{x}{y}} = \sqrt[y]{a^x}$

Question 33

$\frac{4x^{-3}z}{2^{-2}y^4w^{-2}} =$

(A) $\frac{2y^4z}{x^{-3}w^{-2}}$

(B) $\frac{2z}{x^3y^4w^2}$

(C) $\frac{8w^2z}{x^3y^4}$

(D) $\frac{16w^2z}{x^3y^4}$

Question 34

The expression $\dfrac{x^{-2}y^{\frac{1}{2}}}{x^{\frac{1}{3}}y^{-1}}$, where $x > 1$ and $y > 1$, is equivalent to which of the following?

A. $\dfrac{\sqrt{y}}{\sqrt[3]{x^2}}$

B. $\dfrac{y\sqrt{y}}{\sqrt[3]{x^2}}$

C. $\dfrac{y\sqrt{y}}{x\sqrt{x}}$

D. $\dfrac{y\sqrt{y}}{x^2\sqrt[3]{x}}$

Question 35

Which of the following is equivalent to $9^{\frac{3}{4}}$?

A) $\sqrt[3]{9}$

B) $\sqrt[4]{9}$

C) $\sqrt{3}$

D) $3\sqrt{3}$

Question 36

If $\frac{\sqrt{x^5}}{\sqrt[3]{x^4}} = x^{\frac{a}{b}}$ for all positive values of x, what is the value of $\frac{a}{b}$?

Question 37

If $3x - y = 12$, what is the value of $\frac{8^x}{2^y}$?

A) 2^{12}

B) 4^4

C) 8^2

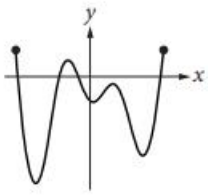
D) The value cannot be determined from the information given.

Functions

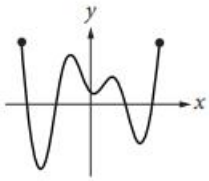
Question 38

If the function f has five distinct zeros, which of the following could represent the complete graph of f in the xy-plane?

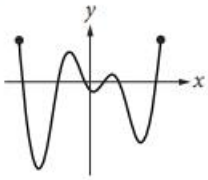
A)



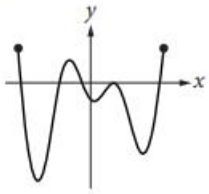
B)



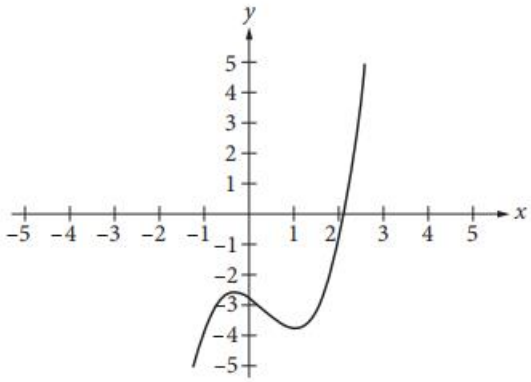
C)



D)



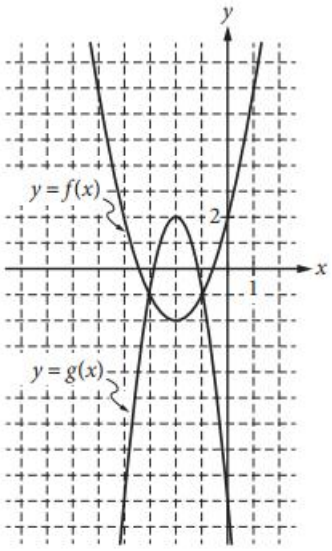
Question 39



The function $f(x) = x^3 - x^2 - x - \frac{11}{4}$ is graphed in the xy-plane above. If k is a constant such that the equation $f(x) = k$ has three real solutions, which of the following could be the value of k ?

A) 2
B) 0
C) −2
D) −3

Question 40



Graphs of the functions f and g are shown in the xy-plane above. For which of the following values of x does $f(x) + g(x) = 0$?

A) -3

B) -2

C) -1

D) 0

Question 41

In the xy-plane, the graph of function f has x-intercepts at -3, -1, and 1. Which of the following could define f ?

A) $f(x) = (x-3)(x-1)(x+1)$

B) $f(x) = (x-3)(x-1)^2$

C) $f(x) = (x-1)(x+1)(x+3)$

D) $f(x) = (x+1)^2(x+3)$

Question 42

For $x > 0$, the function f is defined as follows:

$f(x)$ equals 201% of x

Which of the following could describe this function?

A) Decreasing exponential

B) Decreasing linear

C) Increasing exponential

D) Increasing linear

Question 43

The population of mosquitoes in a swamp is estimated over the course of twenty weeks, as shown in the table.

Time (weeks)	Population
0	100
5	1,000
10	10,000
15	100,000
20	1,000,000

Which of the following best describes the relationship between time and the estimated population of mosquitoes during the twenty weeks?

A) Increasing linear
B) Decreasing linear
C) Exponential growth
D) Exponential decay

Question 44

The world's population has grown at an average rate of 1.9 percent per year since 1945. There were approximately 4 billion people in the world in 1975. Which of the following functions represents the world's population P, in billions of people, t years since 1975 ? (1 billion = 1,000,000,000)

A) $P(t) = 4(1.019)^t$

B) $P(t) = 4(1.9)^t$

C) $P(t) = 1.19t + 4$

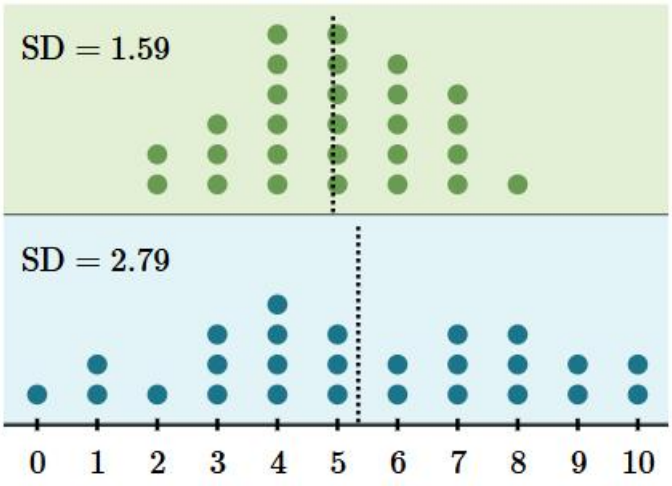
D) $P(t) = 1.019t + 4$

Mean - Mode - Median - Standard Deviation - Probability

Standard Deviation

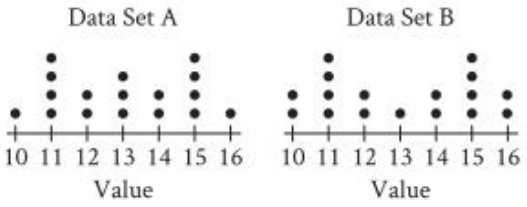
Standard deviation measures the **spread** of a data distribution. **The more spread out a data distribution is, the greater its standard deviation.**

For example, the blue distribution on bottom has a greater standard deviation (SD) than the green distribution on top:



Question 45

The dot plots represent the distributions of values in data sets A and B.



Which of the following statements must be true?

I. The median of data set A is equal to the median of data set B.

II. The standard deviation of data set A is equal to the standard deviation of data set B.

A) I only

B) II only

C) I and II

D) Neither I nor II

Question 46

Number of States with 10 or More Electoral Votes in 2008

Electoral votes	Frequency
10	4
11	4
12	1
13	1
15	3
17	1
20	1
21	2
27	1
31	1
34	1
55	1

In 2008, there were 21 states with 10 or more electoral votes, as shown in the table above. Based on the table, what was the median number of electoral votes for the 21 states?

A) 13

B) 15

C) 17

D) 20

Question 47

Ages of 20 Students Enrolled in a College Class

Age	Frequency
18	6
19	5
20	4
21	2
22	1
23	1
30	1

The table above shows the distribution of ages of the 20 students enrolled in a college class. Which of the following gives the correct order of the mean, median, and mode of the ages?

A) mode < median < mean

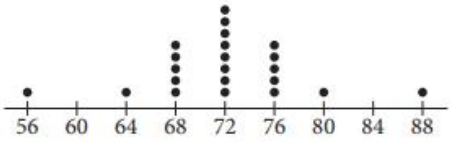
B) mode < mean < median

C) median < mode < mean

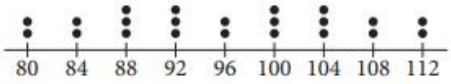
D) mean < mode < median

Question 48

The 22 students in a health class conducted an experiment in which they each recorded their pulse rates, in beats per minute, before and after completing a light exercise routine. The dot plots below display the results.



Beats per minute before exercise



Beats per minute after exercise

Let s_1 and r_1 be the standard deviation and range, respectively, of the data before exercise, and let s_2 and r_2 be the standard deviation and range, respectively, of the data after exercise. Which of the following is true?

A) $s_1 = s_2$ and $r_1 = r_2$

B) $s_1 < s_2$ and $r_1 < r_2$

C) $s_1 > s_2$ and $r_1 > r_2$

D) $s_1 \neq s_2$ and $r_1 = r_2$

Probability

Question 49

The table summarizes the distribution of age and assigned group for **90** participants in a study.

	0–9 years	10–19 years	20+ years	Total
Group A	7	14	9	30
Group B	6	4	20	30
Group C	17	12	1	30
Total	30	30	30	90

One of these participants will be selected at random. What is the probability of selecting a participant from group A, given that the participant is at least **10** years of age? (Express your answer as a decimal or fraction, not as a percent.)

Question 50

Sample	Percent in favor	Margin of error
A	52%	4.2%
B	48%	1.6%

The results of two random samples of votes for a proposition are shown above. The samples were selected from the same population, and the margins of error were calculated using the same method. Which of the following is the most appropriate reason that the margin of error for sample A is greater than the margin of error for sample B?

A. Sample A had a smaller number of votes that could not be recorded.

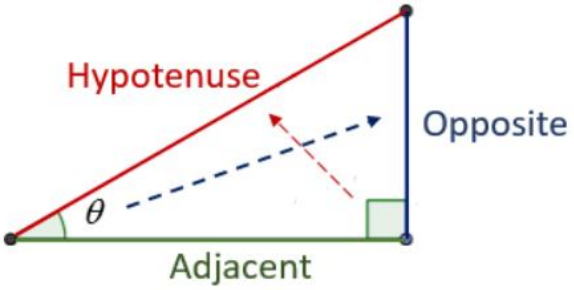
B. Sample A had a higher percent of favorable responses.

C. Sample A had a larger sample size.

D. Sample A had a smaller sample size.

TRIGONOMETRY

SOHCAHTOA



SOH $\quad \sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$

CAH $\quad \cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$

TOA $\quad \tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$

What is SOH-CAH-TOA?

SOHCAHTOA is a mnemonic to help us remember the trigonometric ratios.

Sine = Opposite/Hypotenuse (SOH)

Cosine = Adjacent/Hypotenuse (CAH)

Tangent = Opposite/Adjacent (TOA)

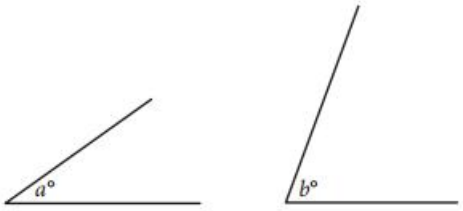
Question 51

Triangle PQR has right angle Q. If $\sin R = \dfrac{4}{5}$, what is the value of $\tan P$?

Question 52

In a right triangle, one angle measures $x°$, where $\sin x° = \frac{4}{5}$. What is $\cos(90° - x°)$?

Question 53



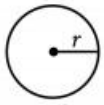
Note: Figures not drawn to scale.

The angles shown above are acute and $\sin(a°) = \cos(b°)$. If $a = 4k - 22$ and $b = 6k - 13$, what is the value of k ?

A) 4.5
B) 5.5
C) 12.5
D) 21.5

GEOMETRY

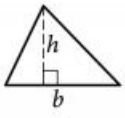
REFERENCE



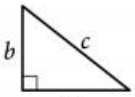
$A = \pi r^2$
$C = 2\pi r$



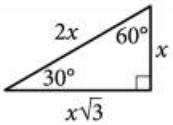
$A = \ell w$

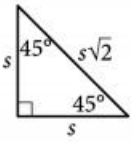


$A = \frac{1}{2}bh$

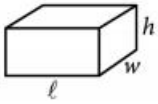


$c^2 = a^2 + b^2$

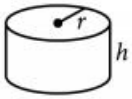




Special Right Triangles



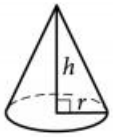
$V = \ell wh$



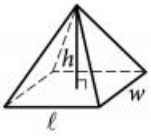
$V = \pi r^2 h$



$V = \frac{4}{3}\pi r^3$



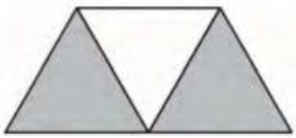
$V = \frac{1}{3}\pi r^2 h$



$V = \frac{1}{3}\ell wh$

The number of degrees of arc in a circle is 360.
The number of radians of arc in a circle is 2π.
The sum of the measures in degrees of the angles of a triangle is 180.

Question 54



A graphic designer is creating a logo for a company. The logo is shown in the figure above. The logo is in the shape of a trapezoid and consists of three congruent equilateral triangles. If the perimeter of the logo is 20 centimeters, what is the combined area of the shaded regions, in square centimeters, of the logo?

A) $2\sqrt{3}$

B) $4\sqrt{3}$

C) $8\sqrt{3}$

D) 16

Question 55

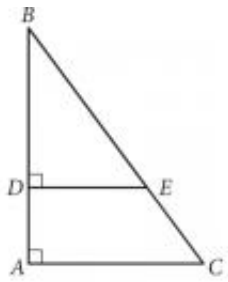
A right triangle has legs with lengths of 24 centimeters and 21 centimeters. If the length of this triangle's hypotenuse, in centimeters, can be written in the form $3\sqrt{d}$, where d is an integer, what is the value of d ?

Question 56

A cube has an edge length of 34 inches. A solid sphere with a radius of 17 inches is inside the cube, such that the sphere touches the center of each face of the cube. To the nearest cubic inch, what is the volume of the space in the cube not taken up by the sphere?

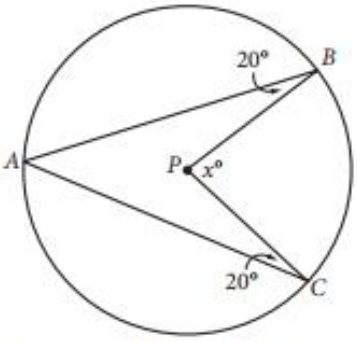
A) 14,576
B) 16,500
C) 18,735
D) 21,018

Question 57



In the figure above, $\tan B = \frac{3}{4}$. If $BC = 15$ and $DA = 4$, what is the length of $\overline{DE}$?

Question 58



Point P is the center of the circle in the figure above.
What is the value of x ?

WORDY PROBLEMS

Question 59

An online store receives customer satisfaction ratings between 0 and 100, inclusive. In the first 10 ratings the store received, the average (arithmetic mean) of the ratings was 75. What is the least value the store can receive for the 11th rating and still be able to have an average of at least 85 for the first 20 ratings?

Question 60

Species of tree	Growth factor
Red maple	4.5
River birch	3.5
Cottonwood	2.0
Black walnut	4.5
White birch	5.0
American elm	4.0
Pin oak	3.0
Shagbark hickory	7.5

One method of calculating the approximate age, in years, of a tree of a particular species is to multiply the diameter of the tree, in inches, by a constant called the growth factor for that species. The table above gives the growth factors for eight species of trees. If a white birch tree and a pin oak tree each now have a diameter of 1 foot, which of the following will be closest to the difference, in inches, of their diameters 10 years from now? (1 foot = 12 inches)

A. 1.0

B. 1.2

C. 1.3

D. 1.4